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TCHEBYSCHEFF VERSUS FARAN-HILLS FOR ARRAY GAINS AND INTENSITY P--ETC(U)

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U. S. NAVY UNDERWATER SOUND LABORATORY
FORT TRUMBULL, NEW LONDON, CONNECTICUT

(6) TCHEBYSCHEFF VS. FARAN-HILLS
FOR ARRAY GAINS AND INTENSITY PATTERNS

by

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(14) USL Problem No.
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(12) 17 p.

INTRODUCTION

Two problems in array design are: (1) increasing array gain and, (2) optimizing the intensity pattern of an array. It is the purpose of this memorandum to review the Faran-Hills method of optimizing array gain, to describe the Dolph-Tchebyscheff procedure and Riblet's extension of the method for optimizing the field pattern of a broadside looking array, and ~~thirdly~~ to compare the resulting patterns and array gains of the two methods.

DOLPH-TCHEBYSCHEFF PROCEDURE FOR OPTIMIZING FIELD PATTERNS

To compute the field patterns of broadside, linear arrays with in phase equally spaced omnidirectional receivers [1], [2], let the center of the array be the reference point and let

d = the distance between elements,

λ = the wave length,

A_k = the amplitude shading factor in the k^{th} element,

and θ = the angle from the normal of the array to the direction from which a plane wave arrives.

The shading factors of the array are arranged for n elements, as

$$\underset{A_1}{o}\ \underset{A_2}{o}\ \underset{A_3}{o}\ \dots\ \underset{A_n}{o}$$

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The field patterns are then given by

$$E_{n\,even} = E_{ne} = 2\sum_{k=0}^{\frac{n}{2}-1} A_k \cos\left\{\frac{2k+1}{2}\left(2\pi\frac{d}{\lambda}\right)\sin\theta\right\} \tag{1}$$

and

$$E_{n\,odd} = E_{no} = 2\sum_{k=0}^{\frac{n-1}{2}} A_k \cos\left\{k\left(2\pi\frac{d}{\lambda}\right)\sin\theta\right\} \tag{2}$$

Let $\psi = 2\pi\frac{d}{\lambda}\sin\theta$, so that

$$E_{ne} = 2\sum_{k=0}^{\frac{n}{2}-1} A_k \cos\left\{(2k+1)\frac{\psi}{2}\right\}, \tag{3}$$

and

$$E_{no} = 2\sum_{k=0}^{\frac{n-1}{2}} A_k \cos\left\{2k\frac{\psi}{2}\right\} \tag{4}$$

C. L. Dolph[3], by making the proper substitutions, rewrites the field pattern equations as polynomials in x with the coefficients made up of the amplitude shading factors.

By de Moivre's theorem,

$$\cos m\frac{\psi}{2} + j\sin\frac{\psi}{2} = \left(\cos\frac{\psi}{2} + j\sin\frac{\psi}{2}\right)^m, \text{ so that}$$

$$\cos m\frac{\psi}{2} = \mathrm{Re}\left(\cos\frac{\psi}{2} + j\sin\frac{\psi}{2}\right)^m, \text{ or}$$

$$\cos m\frac{\psi}{2} = \left(\cos\frac{\psi}{2}\right)^m - \frac{m(m-1)}{2!}\left(\cos\frac{\psi}{2}\right)^{m-2}\left(\sin\frac{\psi}{2}\right)^2 + \frac{m(m-1)(m-2)(m-3)}{4!}\left(\cos\frac{\psi}{2}\right)^{m-4}\left(\sin\frac{\psi}{2}\right)^4 - \cdots \tag{5}$$

Changing the sines to cosines, (5) becomes for particular values of m,

$$\cos m\frac{\psi}{2} = \begin{cases} 1 & \text{for } m=0, \\ \cos\frac{\psi}{2} & \text{for } m=1, \\ 2\cos^2\frac{\psi}{2} - 1 & \text{for } m=2, \\ 4\cos^3\frac{\psi}{2} - 3\cos\frac{\psi}{2} & \text{for } m=3, \\ 8\cos^4\frac{\psi}{2} - 8\cos^2\frac{\psi}{2} + 1 & \text{for } m=4. \end{cases} \tag{6}$$

Substituting for $\cos m\frac{\psi}{2}$ into (1) and (2) and then letting

$$x = \cos\frac{\psi}{2}, \tag{7}$$

$$E_{ne}(x) = 2\left\{A_0x + A_1(4x^3 - 3x) + A_2(16x^5 - 20x^3 + 5x) + \cdots\right\} \tag{8}$$

$$E_{no}(x) = 2\left\{A_0 + A_1(2x^2 - 1) + A_2(8x^4 - 8x^2 + 1) + A_3(32x^6 - 48x^4 + 18x^2 - 1) + \cdots\right\} \tag{9}$$

Collecting terms,

$$E_{ne} = 2\left\{x(A_0 - 3A_1 + 5A_2 + \cdots) + x^3(4A_1 - 20A_2 + \cdots) + x^5(16A_2 + \cdots) + \cdots\right\} \tag{10}$$

and

$$E_{no} = 2\left\{(A_0 - A_1 + A_2 - A_3 + \cdots) + x^2(2A_1 - 8A_2 + 18A_3 + \cdots) + x^4(8A_2 - 48A_3 + \cdots) + \cdots\right\} \tag{11}$$

For n elements, the degree of the resulting equation will be $n-1$. Dolph chose to equate the field pattern equations to the Tchebyscheff polynomials because this results in an optimum pattern in the sense that for a given beam width the minor lobe level is minimum and vice versa.

The Tchebyscheff polynomials are defined as $T_m(x) = \cos m\frac{\psi}{2}$. For particular values of m,

$$T_m(x) = \begin{cases} 1 & \text{for } m=0 \\ x & \text{for } m=1 \\ 2x^2-1 & \text{for } m=2 \\ 4x^3-3x & \text{for } m=3 \\ 8x^4-8x^2+1 & \text{for } m=4 \\ 16x^5-20x^3+5x & \text{for } m=5 \\ 32x^6-48x^4+18x^2-1 & \text{for } m=6 \\ 64x^7-112x^5+56x^3-7x & \text{for } m=7 \end{cases} \tag{12}$$

All Tchebyscheff polynomials have the following properties, among others.

1. All pass thru the point (1,1).
2. For $-1 \le x \le 1$, $|T_m(x)| \le 1$ for all m,

3. All roots are in the interval (-1, 1), and
4. All maxima and minima occur in the interval (-1, 1) and alternate between -1 and 1.

The point (x_0, R) in Figure 1 will be made to correspond to the main lobe maximum while the maxima, minima and zeros will correspond to the minor lobes and nulls of the field pattern.

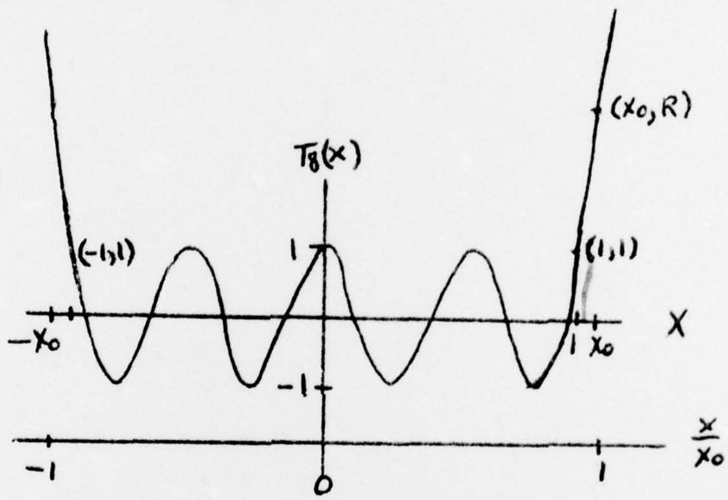

figure 1

For another polynomial of like degree to have the same beam width i.e., pass thru (x_0, R) and the largest zero and also have a smaller minor lobe level over the range $(-1, 1)$, it would have to intersect the Tchebyscheff polynomial in $n+1$ places. The fact that any two n^{th} degree polynomials which interesect in at least $n+1$ places must coincide, proves that the Tchebyscheff polynomials are indeed optimum.

Define $R = \frac{\text{main lobe maximum}}{\text{side lobe level}}$. Note that for the Tchebyscheff polynomials the denominator is always equal to one so that R = main lobe maximum. x_0 will usually be equal to or greater than one (and can be found by solving $T_{n-1}(x_0) = R$) but equation (7) restricts x to the range $-1 \leq x \leq 1$, so Dolph subjects the abscissa to the linear transformation $w = x/x_0$ (see Figure 1) and then equates $E_n\left(\frac{x}{x_0}\right)$ as given by equation (10) or (11) to $T_{n-1}(x)$. Solving for the coefficients of x gives the amplitude shading factors. An example is given in the appendix. Riblet[4] shows that this method is restricted to spacings where $\frac{d}{\lambda} \geq \frac{1}{2}$. From (7) $x = (\cos \frac{\pi d}{\lambda} \sin \theta)$, and as θ ranges from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, the observable pattern varies from $a = (\cos \frac{\pi d}{\lambda})$ to one and back to a again. As $\frac{d}{\lambda}$

becomes less than 1/2, a becomes greater than zero. There are now polynomials of like degree which give lower side lobe levels for the same beam width in the range from a to one and Dolph's procedure no longer produces an optimum pattern. Figure 2 shows a polynomial with better side lobe and beamwidth characteristics than is obtainable by Dolph's method.

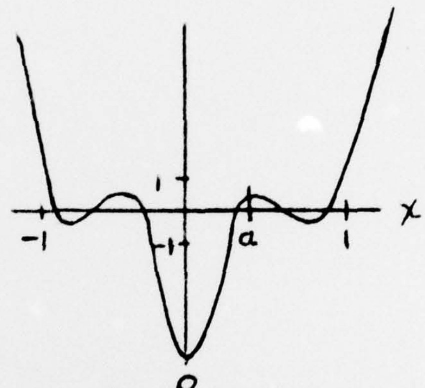

figure 2

Riblet shows that when there are an odd number of elements he can make the substitution $x = (\cos \frac{2\pi d}{\lambda} \sin\theta)$ so that $E_n(x)$ is given by a polynomial of degree $\frac{n-1}{2}$ instead of $n-1$ as would be the case in Dolph's method. Now, by the more general transformation $bx+c$ performed on $T_{\frac{n-1}{2}}(x)$, he finds b and c so that the abscissa from $x=-1$ to $x=x_0$ of $T_{\frac{n-1}{2}}(x)$ is squeezed between a and one in $T_{\frac{n-1}{2}}(bx+c)$. This procedure results in the optimum pattern. An example of this procedure is also given in the appendix. When n is even, the substitution $x = (\cos \frac{2\pi d}{\lambda} \sin\theta)$ cannot be made so there is no equivalent procedure to optimize the pattern in this case.

FARAN-HILLS PROCEDURE FOR OPTIMIZING ARRAY GAIN

Define array gain as

$$G = \frac{\overline{S_A^2}}{\overline{N_A^2}} \Big/ \frac{\overline{S_e^2}}{\overline{N_e^2}}$$

where $\overline{S_e^2}$ is the mean square output of the signal alone in one element.
$\overline{N_e^2}$ is the mean square output of the noise alone in one element.
$\overline{S_A^2}$ is the mean square output of the signal alone from the array.
$\overline{N_A^2}$ is the mean square output of the noise alone from the array.

Let A_K be the amplitude shading factor of the k^{th} element in an additive array. Then

$$G = \frac{\overline{\left(\sum_{1}^{n} A_K s_K\right)^2}}{\overline{\left(\sum_{1}^{n} A_K n_K\right)^2}} \bigg/ \frac{\overline{S_e^2}}{\overline{Ne^2}}$$

where s_K is the signal input at the k^{th} element and
n_K is the noise input at the K^{th} element.

Assume that the signal is perfectly coherent, that the noise is homogeneous and stationary in the wide sense, that $\overline{n_K n_L} = \overline{N^2} \rho_{KL}$, where ρ_{KL} is the spatial correlationbetween the K^{th} and L^{th} elements and let $\sum_{K=1}^{n} A_K = 1$. Then

$$G = \frac{1}{\sum_{K}^{n} \sum_{L}^{n} A_K A_L \rho_{KL}} .$$

Faran and Hills[5] maximize G by themethod of Lagrange's undetermined multipliers giving the following set of $k+1$ equations.

$$R = \sum A_K - 1 = 0$$

$$\frac{\partial G}{\partial A_K} + \mu \frac{\partial R}{\partial A_K} = 0, \quad K = 1, 2, \ldots, n$$

Solving the equations gives the optimum A_K for the maximum array gain. For more details of this procedure see reference (6).

PROCEDURE AND RESULTS

A five element array was used to compare the Tchebyscheff and Faran-Hills methods since an odd number of elements are required for the cases where $\frac{d}{\lambda} < \frac{1}{2}$, and because the Faran-Hills array gain, amplitude shading coefficients and corresponding patterns for the five element array already were available for spacings of $\frac{d}{\lambda} = \frac{1}{8}, \frac{2}{8}, \ldots, 1$. These included both assumptions of an isotropic noise field and a directional noise field, where directional means there are independent noise sources, equally distributed on a large circular area on the surface of the ocean, radiating with an intensity proportional to the $\cos^2\alpha$, α being the angle between the direction of radiation and the perpendicular to the surface of the ocean.

A computer program was written to take the ratios of the main lobe maximum to the largest side lobe maximum from the Faran-Hills results for a single frequency and compute the corresponding Tchebyscheff shading factors and intensity patterns. (Dolph's procedure for $\frac{d}{\lambda} > \frac{1}{2}$ and Riblet's procedure for $\frac{d}{\lambda} \leq \frac{1}{2}$). These Tchebyscheff shading factors were then used to compute the array gain in the same noise field as the corresponding Faran-Hills results. Figure 4 is a table comparing the amplitude shading coefficients, array gains and beam width at the 3 db down points for the same noise fields, spacings and side lobe levels. Figure 5 - 8 are the intensity patterns as obtained from the Faran-Hills and Tchebyscheff procedures.

SUMMARY AND CONCLUSIONS

A linear, five element, evenly spaced array has been used to compare, on a computer, array gains, amplitude shading coefficients and intensity patterns with identical side lobe levels as obtained from the Faran-Hills procedure which maximizes array gains using noise field characteristics and the Tchebyscheff procedures which optimizes the intensity patterns but considers no noise field. In order to compare the array gains properly the Tchebyscheff array gains were computed by assuming the appropriate noise fields.

For the cases of $\frac{d}{\lambda} = \frac{1}{8}, \frac{2}{8}, \ldots \frac{6}{8}$, there are little differences in the shading coefficients, consequently there are little differences in the array gains and intensity patterns. The Faran-Hills procedure results in somewhat greater gains but also somewhat wider beam widths. The maximum difference in array gains is approximately 1/2 db and the maximum difference in beam widths is about 1 degree at the 3 db downpoint. For $\frac{d}{\lambda} = \frac{7}{8}$ and 1, the differences are greater.

APPENDIX

1. An Example of the Dolph-Tchebyscheff Method[1]

Let $n = 8,\ \frac{d}{\lambda} = \frac{1}{2}$
and
$R = 20.$

From equation (10) dropping the factor 2 (we're interested only in the relative field pattern),

$$E_8 = (A_4 - 3A_3 + 5A_2 - 7A_1)x + (4A_3 - 20A_2 + 56A_1)x^3 + (16A_2 - 112A_1)x^5 + 64A_1x^7.$$

From $T_7(x) = 20$, $x_0 \cong 1.15$ Applying the linear transformation $x/1.15$ $E_8(x/1.15)$ is set equal to $T_7(x)$ and is solved for the A_i, $i=1,2,3$. $E_8(x) = T_7(1.15x)$ is equivalent to $E_8(x/1.15) = T_7(x)$.

$$T_7(1.15x) = 64(1.15x)^7 - 112(1.15x)^5 + 56(1.15x)^3 - 7(1.15x).$$

Equating coefficients,

$$64 A_1 x^7 = 64(1.15)^7 x^7 \implies A_1 = 2.66,$$
$$(16A_2 - 112A_1)x^5 = -112(1.15)^5 x^5 \implies A_2 = 4.56,$$
$$(4A_3 - 20A_2 + 56A_1)x^3 = 56(1.15)^3 x^3 \implies A_3 = 6.82,$$
$$(A_4 - 3A_3 + 5A_2 - 2A_1)x = -7(1.15)x \implies A_4 = 8.25.$$

Normalizing,

$$A_4 = A_5 = 3.1,$$
$$A_3 = A_6 = 2.6,$$
$$A_2 = A_7 = 1.7 \text{ and}$$
$$A_1 = A_8 = 1.0.$$

To get the pattern, $x = x_0(\cos\frac{\pi d}{2}\sin\theta)$ and as θ goes from $-\pi/2$ to 0 to $\pi/2$, the square of the values of $E_8(x)$ is the intensity of the direction θ .

2. An Example of Riblet's Method for $\frac{d}{\lambda} \leq \frac{1}{2}$ [4]

Let $n = 7$, $\frac{d}{\lambda} = \frac{1}{3}$,

, and $R = 9$

The equation of the pattern is

$$E_7 = A_4 + A_3\left(\cos\frac{2\pi}{3}\sin\theta\right) + A_2\left(\cos\frac{4\pi}{3}\sin\theta\right) + A_1\left(\cos\frac{6\pi}{3}\sin\theta\right).$$

Let $x = \cos\left(\frac{2\pi}{3}\sin\theta\right)$ (1)

$a = \cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$

and $E_7 = A_4 + A_3 x + A_2(2x^2 - 1) + A_1(4x^3 - 3x)$ (2)

The corresponding Tchebyscheff polynomial is $T_3(x) = 4x^3 - 3x$,

and $T_3(x_0) = 9 \Longrightarrow x_0 = 3/2$

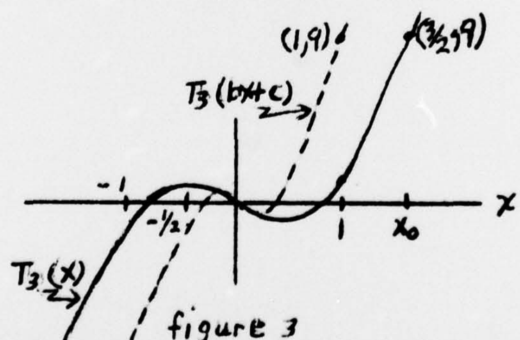

figure 3

Applying for the transformation $bx+c$ so that all x values from -1 to $3/2$ will be squeezed between $-1/2$ and 1 , let

$$b\left(-\tfrac{1}{2}\right) + c = -1 \quad \text{and}$$

$$b(1) + c = 3/2$$

Therefore $b = 5/3$, and

$$c = -1/6$$

$E_7 = A_4 - A_2 + (A_3 - 3A_1)x + 2A_2 x^2 + 4A_1 x^3$ is set equal to $T_3\left(\frac{5x}{3} - \frac{1}{6}\right)$. That is,

$$A_4 - A_2 + (A_3 - 3A_1)x + 2A_2 x^2 + 4A_1 x^3 = 18.5x^3 - 5.55x^2 - 4.45x + .481.$$

Equating coefficients and normalizing,

$$A_4 = 1$$
$$A_3 = A_5 = -4.07$$
$$A_2 = A_6 = 1.19$$
$$A_1 = A_7 = -2.00$$

To plot the pattern, substitute $x = \cos\left(\frac{2\pi d}{\lambda}\sin\theta\right)$ in (2). The square of the values of E_7 gives the intensity in the direction θ. To return to the original form, replace x with $2x^2 - 1$. The resulting polynomial is similar to the one shown in Figure 2.

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John P. Beam by CJB
JOHN P. BEAM
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(4) Discussion on "A Current Distribution for Broadside Arrays which Optimizes the Relationship Between Beam Width and Side-Lobe Level", by Henry J. Riblet, Proc. Ire., Vol. 35, 1947, pp 489-492

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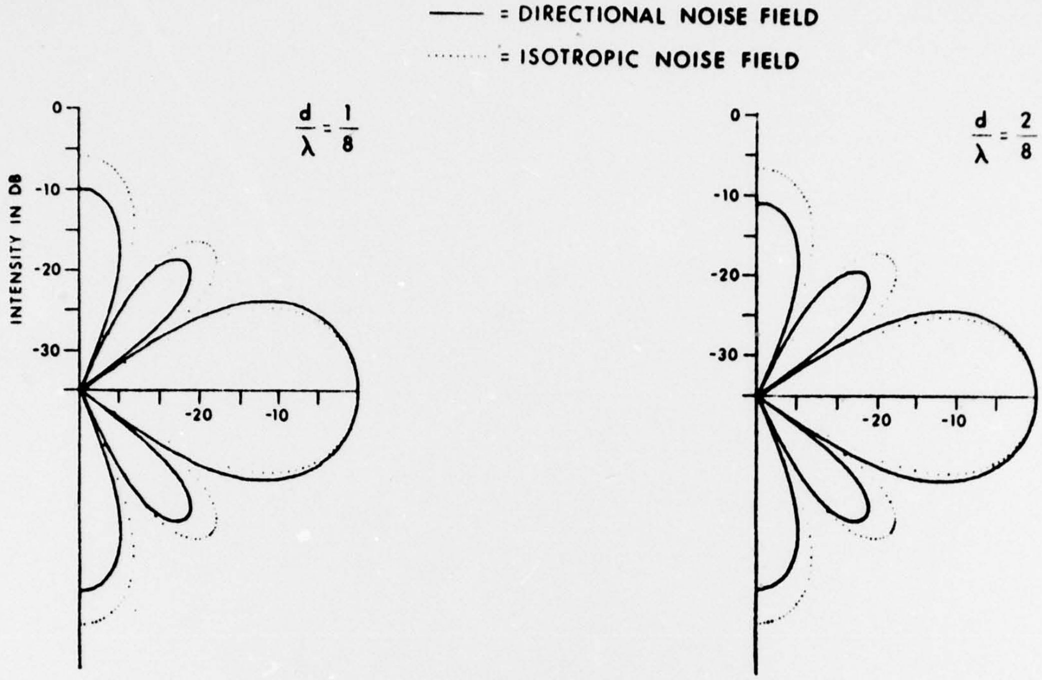
No.	d/λ	Side lobe level	SHADING Faran - Hills A_1	A_2	A_3	COEFFICIENTS Dolph-Tchebyscheff A_1	A_2	A_3	Beam Width 3 db down point F-H	T	Array Gain (db) F-H	T
D_{1r}	$\frac{1}{8}$	-7.8	9.56	-31.48	44.84	11.14	-36.87	52.47	15.9°	14.9°	9.7	9.3
I_{S0}	$\frac{1}{8}$	-5.6	12.08	-40.41	57.61	14.88	-49.98	71.21	14.6°	13.2°	5.5	5.0
D_{1r}	$\frac{2}{8}$	-10.9	0.74	-1.22	1.80	0.89	-1.43	2.07	14.9°	14.2°	10.2	9.9
I_{S0}	$\frac{2}{8}$	-6.6	1.00	-1.73	2.46	1.19	-2.11	2.84	13.7°	12.6°	5.8	5.6
D_{1r}	$\frac{3}{8}$	-13.2	0.25	0.08	0.34	0.27	0.07	0.32	13.4°	13.0°	11.1	11.0
I_{S0}	$\frac{3}{8}$	-8.5	0.32	-0.00	0.37	0.36	-0.03	0.34	12.2°	11.6°	6.3	6.2
D_{1r}	$\frac{1}{2}$	-16.1	0.16	0.22	0.24	0.17	0.21	0.24	11.3°	11.2°	12.5	12.4
I_{S0}	$\frac{1}{2}$	-12.1	0.20	0.20	0.20	0.21	0.19	0.21	10.3°	10.3°	7.0	7.0
D_{1r}	$\frac{5}{8}$	-16.6	0.15	0.23	0.24	0.16	0.21	0.25	9.1°	9.0°	14.3	14.0
I_{S0}	$\frac{5}{8}$	-12.1	0.19	0.22	0.19	0.21	0.19	0.21	8.4°	8.2°	7.9	7.8
D_{1r}	$\frac{6}{8}$	-21.5	0.13	0.23	0.28	0.13	0.23	0.28	8.0°	8.0°	15.5	15.5
I_{S0}	$\frac{6}{8}$	-15.0	0.17	0.21	0.23	0.18	0.21	0.24	7.3°	7.2°	8.5	8.5
D_{1r}	$\frac{7}{8}$	-5.5	0.17	0.21	0.23	0.32	0.12	0.12	6.3°	4.9°	13.6	8.6
I_{S0}	$\frac{7}{8}$	-5.9	0.18	0.21	0.22	0.31	0.12	0.13	6.1°	5.0°	8.9	7.2
D_{1r}	1	0	0.20	0.20	0.20	0.50	0.0	0.0	5.1°	3.0°	7.0	3.0
I_{S0}	1	0	0.20	0.20	0.20	0.50	0.0	0.0	5.3°	3.3°	7.0	3.0

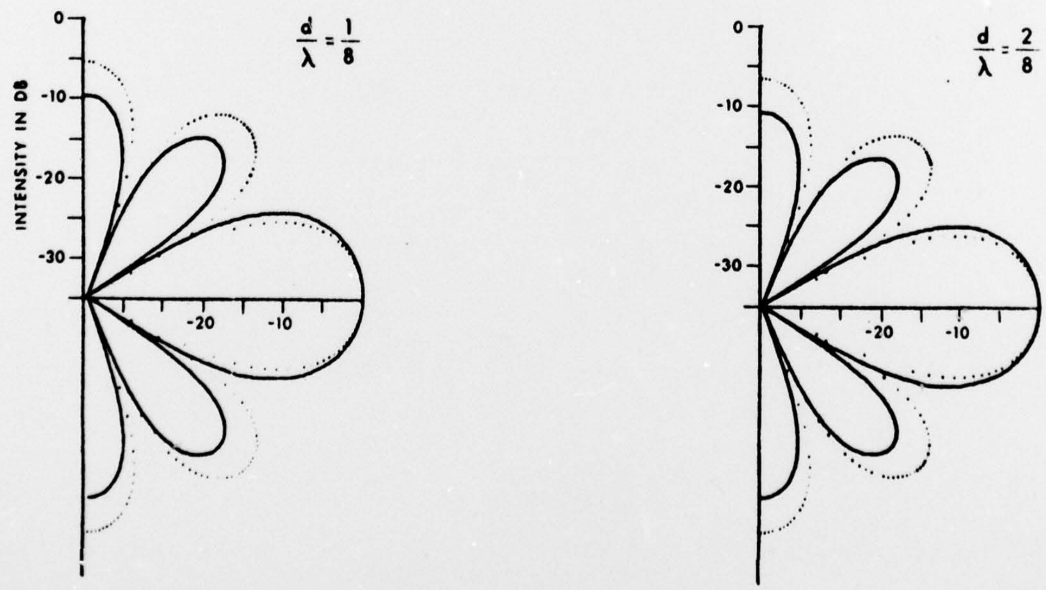
COMPARISON OF FARAN-HILLS SHADING VERSUS TCHEBYSCHEFF SHADING

Shading factors are normalized so that $A_1 + A_2 + A_3 + A_4 + A_5 = 1$.

FIGURE 4

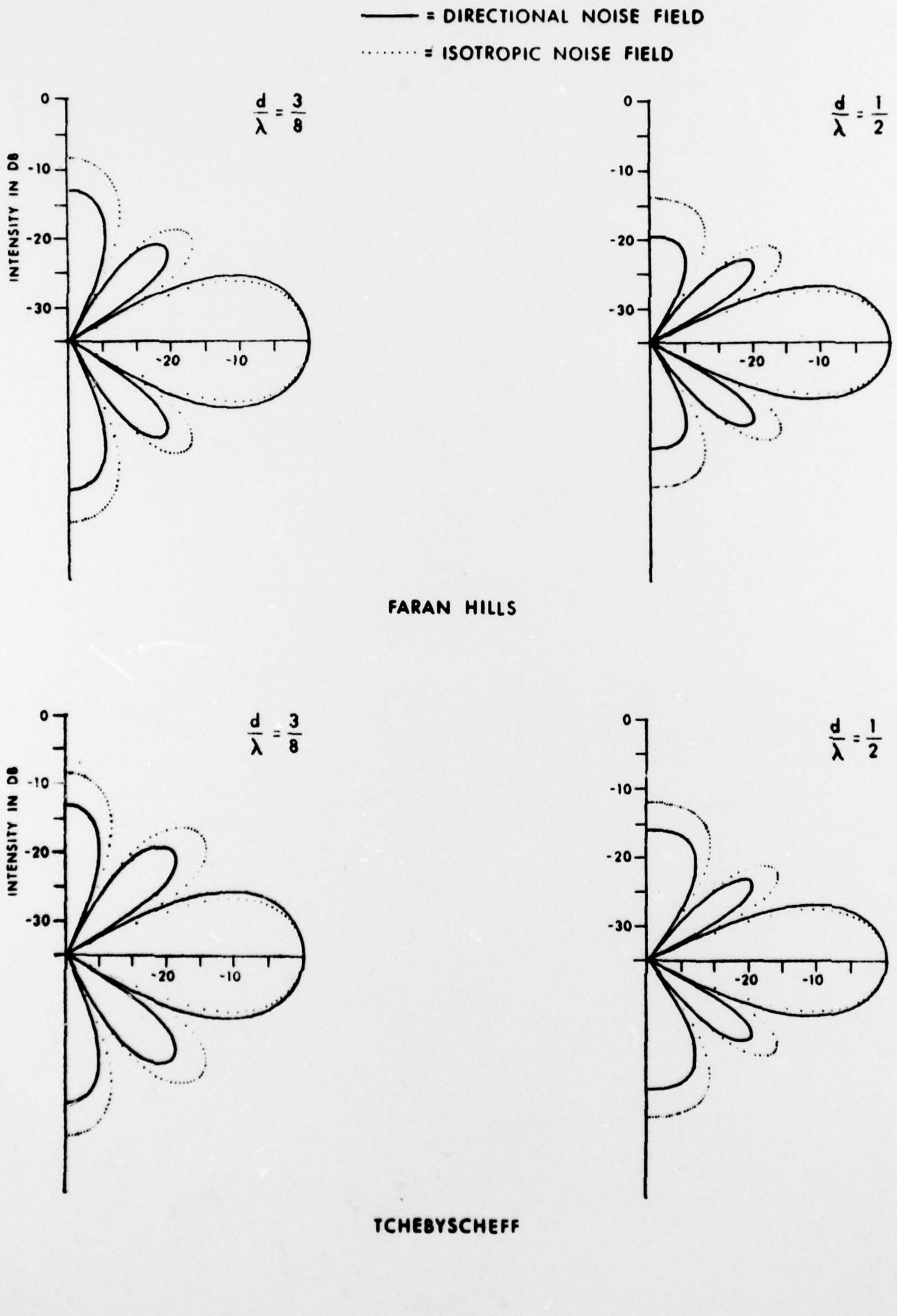


FARAN HILLS



TCHEBYSCHEFF

FIGURE 5



= DIRECTIONAL NOISE FIELD
= ISOTROPIC NOISE FIELD
d/λ = 3/8
d/λ = 1/2
INTENSITY IN DB
0
-10
-20
-30
-20
-10
FARAN HILLS
TCHEBYSCHEFF

FIGURE 6

= DIRECTIONAL NOISE FIELD
= ISOTROPIC NOISE FIELD
INTENSITY IN DB
0
-10
-20
-30
-20
-10
d/λ = 5/8
d/λ = 6/8
FARAN HILLS
TCHEBYSCHEFF

FIGURE 7

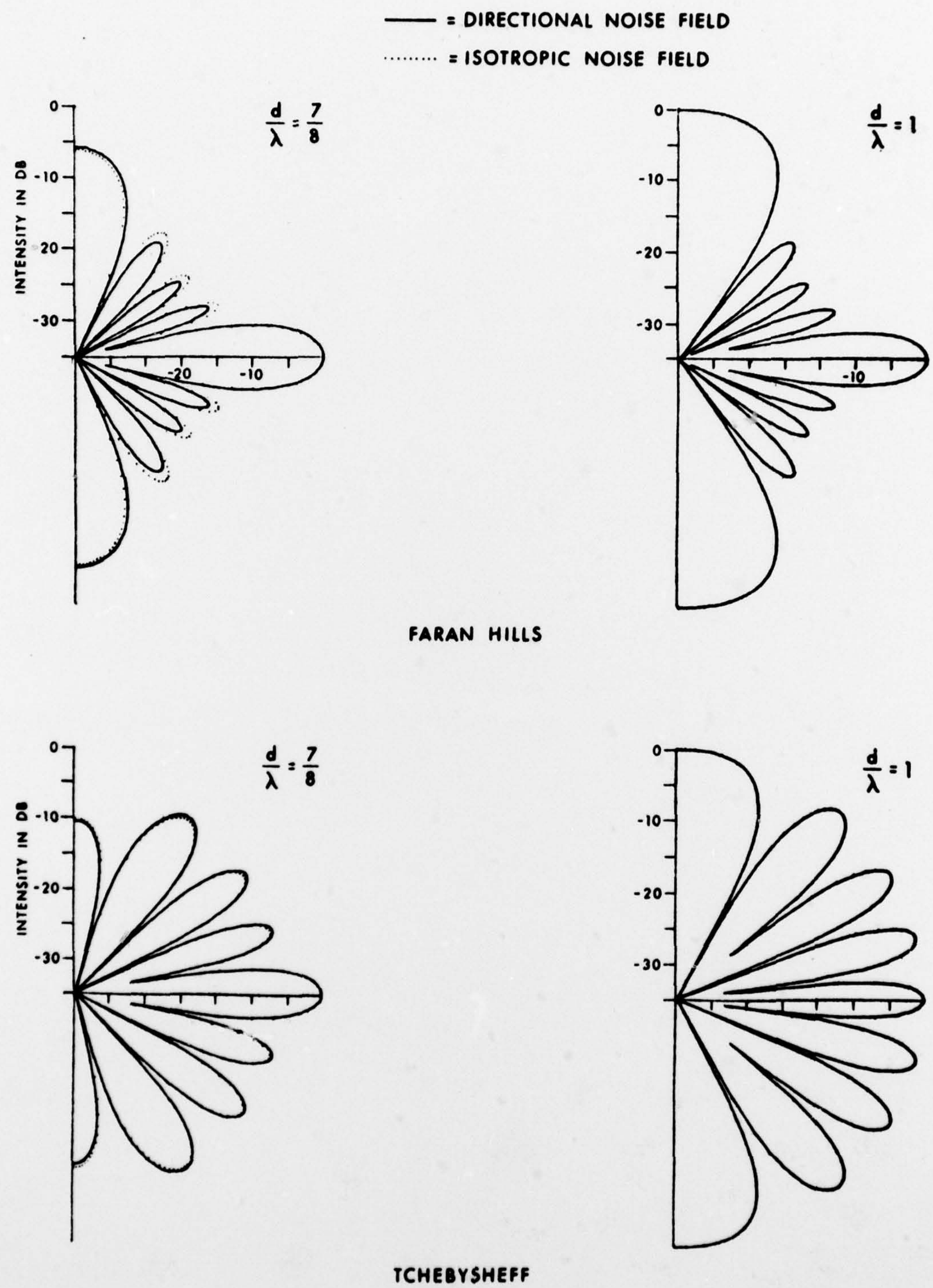

— = DIRECTIONAL NOISE FIELD
······· = ISOTROPIC NOISE FIELD
INTENSITY IN DB
0
-10
-20
-30
d/λ = 7/8
d/λ = 1
-20
-10
FARAN HILLS
TCHEBYSHEFF

FIGURE 8

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